In this lesson, you will study four proportionality theorems. Similar triangles are used to prove each theorem. You are asked to prove the theorems in Exercises 31–33 and 38.

**THEOREMS**

**THEOREM 8.4 Triangle Proportionality Theorem**

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

If \( TU \parallel QS \), then \( \frac{RT}{TQ} = \frac{RU}{US} \).

**THEOREM 8.5 Converse of the Triangle Proportionality Theorem**

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

If \( \frac{RT}{TQ} = \frac{RU}{US} \), then \( TU \parallel QS \).

**EXAMPLE 1 Finding the Length of a Segment**

In the diagram \( AB \parallel ED \),
\( BD = 8 \), \( DC = 4 \), and \( AE = 12 \).
What is the length of \( EC \)?

**SOLUTION**

\[
\frac{DC}{BD} = \frac{EC}{AE}
\]

\[
\frac{4}{8} = \frac{EC}{12}
\]

\[
\frac{4(12)}{8} = EC
\]

\[
6 = EC
\]

So, the length of \( EC \) is 6.
**EXAMPLE 2  Determining Parallels**

Given the diagram, determine whether \( MN \parallel GH \).

**SOLUTION**

Begin by finding and simplifying the ratios of the two sides divided by \( MN \).

\[
\frac{LM}{MG} = \frac{56}{21} = \frac{8}{3} \quad \frac{LN}{NH} = \frac{48}{16} = \frac{3}{1}
\]

Because \( \frac{8}{3} \neq \frac{3}{1} \), \( MN \) is not parallel to \( GH \).

**THEOREMS**

**THEOREM 8.6**

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

If \( r \parallel s \) and \( s \parallel t \), and \( \ell \) and \( m \) intersect \( r, s, \) and \( t \), then \( \frac{UW}{WY} = \frac{VX}{XZ} \).

**THEOREM 8.7**

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

If \( \overline{CD} \) bisects \( \angle ACB \), then \( \frac{AD}{DB} = \frac{CA}{CB} \).

**EXAMPLE 3  Using Proportionality Theorems**

In the diagram, \( \angle 1 \equiv \angle 2 \equiv \angle 3 \), and \( PQ = 9 \), \( QR = 15 \), and \( ST = 11 \). What is the length of \( TU \)?

**SOLUTION**

Because corresponding angles are congruent the lines are parallel and you can use Theorem 8.6.

\[
\frac{PQ}{QR} = \frac{ST}{TU} \quad \text{Parallel lines divide transversals proportionally.}
\]

\[
\frac{9}{15} = \frac{11}{TU} \quad \text{Substitute.}
\]

\[
9 \cdot TU = 15 \cdot 11 \quad \text{Cross product property}
\]

\[
TU = \frac{15(11)}{9} = \frac{55}{3} \quad \text{Divide each side by 9 and simplify.}
\]

So, the length of \( TU \) is \( \frac{55}{3} \), or \( 18\frac{1}{3} \).
EXAMPLE 4  Using Proportionality Theorems

In the diagram, $\angle CAD \cong \angle DAB$. Use the given side lengths to find the length of $DC$.

**SOLUTION**

Since $\overline{AD}$ is an angle bisector of $\angle CAB$, you can apply Theorem 8.7.

Let $x = DC$. Then, $BD = 14 - x$.

\[
\frac{AB}{AC} = \frac{BD}{DC}
\]

Apply Theorem 8.7.

\[
\frac{9}{15} = \frac{14 - x}{x}
\]

Substitute.

\[
9 \cdot x = 15(14 - x)
\]

Cross product property

\[
9x = 210 - 15x
\]

Distributive property

\[
24x = 210
\]

Add 15x to each side.

\[
x = 8.75
\]

Divide each side by 24.

So, the length of $DC$ is 8.75 units.

---

**ACTIVITY**

**Construction**

**Dividing a Segment into Equal Parts (4 shown)**

1. Draw a line segment that is about 3 inches long. Label the endpoints $A$ and $B$. Choose any point $C$ not on $\overline{AB}$. Draw $\overline{AC}$.

![Diagram of step 1](image1)

2. Using any length, place the compass point at $A$ and make an arc intersecting $\overline{AC}$ at $D$. Draw $\overline{GB}$.

![Diagram of step 2](image2)

3. Using the same compass setting, make additional arcs on $\overline{AC}$. Label the points $E$, $F$, and $G$ so that $AD = DE = EF = FG$.

![Diagram of step 3](image3)

4. Draw $\overline{GB}$. Construct a line parallel to $\overline{GB}$ through $D$. Continue constructing parallel lines and label the points as shown. Explain why $AJ = JK = KL = LB$.

![Diagram of step 4](image4)
**EXAMPLE 5**  
*Finding the Length of a Segment*

**BUILDING CONSTRUCTION** You are insulating your attic, as shown. The vertical 2 × 4 studs are evenly spaced. Explain why the diagonal cuts at the tops of the strips of insulation should have the same lengths.

**SOLUTION**
Because the studs $AD$, $BE$, and $CF$ are each vertical, you know that they are parallel to each other. Using Theorem 8.6, you can conclude that $\frac{DE}{EF} = \frac{AB}{BC}$.

Because the studs are evenly spaced, you know that $DE = EF$. So, you can conclude that $AB = BC$, which means that the diagonal cuts at the tops of the strips have the same lengths.

**EXAMPLE 6**  
*Finding Segment Lengths*

In the diagram $KL \parallel MN$. Find the values of the variables.

**SOLUTION**
To find the value of $x$, you can set up a proportion.

$$\frac{9}{13.5} = \frac{37.5 - x}{x}$$

Write proportion.

$13.5(37.5 - x) = 9x$

Cross product property

$506.25 - 13.5x = 9x$

Distributive property

$506.25 = 22.5x$

Add 13.5x to each side.

$22.5 = x$

Divide each side by 22.5.

Since $KL \parallel MN$, $\triangle JKL \sim \triangle JMN$ and $\frac{JK}{JM} = \frac{KL}{MN}$.

$$\frac{9}{13.5 + 9} = \frac{7.5}{y}$$

Write proportion.

$9y = 7.5(22.5)$

Cross product property

$y = 18.75$

Divide each side by 9.
1. Complete the following: If a line divides two sides of a triangle proportionally, then it is __?__ to the third side. This theorem is known as the __?__.

2. In ΔABC, AR bisects ΔCAB. Write the proportionality statement for the triangle that is based on Theorem 8.7.

Determine whether the statement is true or false. Explain your reasoning.

3. \( \frac{FE}{ED} = \frac{FG}{GH} \)

4. \( \frac{FE}{FD} = \frac{FG}{FH} \)

5. \( \frac{EG}{DH} = \frac{EF}{DF} \)

6. \( \frac{ED}{FE} = \frac{EG}{DH} \)

Use the figure to complete the proportion.

7. \( \frac{BD}{BF} = \frac{?}{CG} \)

8. \( \frac{AE}{CE} = \frac{?}{BD} \)

9. \( \frac{?}{GA} = \frac{FD}{FA} \)

10. \( \frac{GA}{?} = \frac{FA}{DA} \)

LOGICAL REASONING Determine whether the given information implies that QS ∥ PT. Explain.

11. 12.

LOGICAL REASONING Use the diagram shown to decide if you are given enough information to conclude that LP ∥ MQ. If so, state the reason.

15. \( \frac{NM}{ML} = \frac{NQ}{QP} \)

16. \( \angle MNQ \equiv \angle LNP \)

17. \( \angle NLP \equiv \angle NMQ \)

18. \( \angle MQN \equiv \angle LPN \)

19. \( \frac{LM}{MN} = \frac{LP}{MQ} \)

20. \( \triangle LPN \sim \triangle MQN \)
Using Proportionality Theorems  Find the value of the variable.

21. \( \frac{9}{a} = \frac{5}{15} \)

22. \( \frac{20}{24} = \frac{12}{c} \)

23. \( \frac{8}{15} = \frac{20}{x} \)

24. \( \frac{8}{12} = \frac{25}{z} \)

Using Algebra  Find the value of the variable.

25. \( \frac{12}{p} = \frac{7}{24} \)

26. \( \frac{21}{q} = \frac{17.5}{33} \)

27. \( \frac{f}{6} = \frac{21}{15} \)

28. \( \frac{14}{1.25y} = \frac{17.5}{7.5} \)

Lot Prices  The real estate term for the distance along the edge of a piece of property that touches the ocean is “ocean frontage.”

29. Find the ocean frontage (to the nearest tenth of a meter) for each lot shown.

30. Critical Thinking  In general, the more ocean frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?
31. **Two-Column Proof** Use the diagram shown to write a two-column proof of Theorem 8.4.

**Given** \[ DE \parallel AC \]

**Prove** \[ \frac{DA}{DB} = \frac{EC}{BE} \]

32. **Paragraph Proof** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 8.6.

**Given** \[ k_1 \parallel k_2, k_2 \parallel k_3 \]

**Prove** \[ \frac{CB}{BA} = \frac{DE}{EF} \]

33. **Paragraph Proof** Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 8.7.

**Given** \[ \angle YXW \cong \angle WXZ \]

**Prove** \[ \frac{YW}{WZ} = \frac{XY}{XZ} \]

**Finding Segment Lengths** Use the diagram to determine the lengths of the missing segments.

34. \[ A \quad 8 \quad 11.9 \quad D \quad 3.5 \quad H \]

35. \[ K \quad 18 \quad 18 \quad L \quad 12 \quad 14 \quad Q \]

**New York City** Use the following information and the map of New York City.

On Fifth Avenue, the distance between E 33rd Street and E 24th Street is about 2600 feet. The distance between those same streets on Broadway is about 2800 feet. All numbered streets are parallel.

36. On Fifth Avenue, the distance between E 24th Street and E 29th Street is about 1300 feet. What is the distance between these two streets on Broadway?

37. On Broadway, the distance between E 33rd Street and E 30th Street is about 1120 feet. What is the distance between these two streets on Fifth Avenue?
38. **Writing** Use the diagram given for the proof of Theorem 8.4 from Exercise 31 to explain how you can prove the Triangle Proportionality Converse, Theorem 8.5.

39. **Multi-Step Problem** Use the diagram shown.
   a. If $DB = 6$, $AD = 2$, and $CB = 20$, find $EB$.
   b. Use the diagram to state three correct proportions.
   c. If $DB = 4$, $AB = 10$, and $CB = 20$, find $CE$.
   d. **Writing** Explain how you know that $\triangle ABC$ is similar to $\triangle DBE$.

40. **Construction** Perform the following construction.
   
   **Given** ▶ Segments with lengths $x$, $y$, and $z$

   **Construct** ▶ A segment of length $p$, such that $\frac{x}{y} = \frac{z}{p}$
   
   *(Hint: This construction is like the construction on page 500.)*

**Mixed Review**

**Using the Distance Formula** Find the distance between the two points. *(Review 1.3)*

- 41. $A(10, 5)$ $B(-6, -4)$
- 42. $A(7, -3)$ $B(-9, 4)$
- 43. $A(-1, -9)$ $B(6, -2)$
- 44. $A(0, 11)$ $B(-5, 2)$
- 45. $A(0, -10)$ $B(4, 7)$
- 46. $A(8, -5)$ $B(0, 4)$

**Using the Distance Formula** Place the figure in a coordinate plane and find the requested information. *(Review 4.7)*

- 47. Draw a right triangle with legs of 12 units and 9 units. Find the length of the hypotenuse.
- 48. Draw a rectangle with length 16 units and width 12 units. Find the length of a diagonal.
- 49. Draw an isosceles right triangle with legs of 6 units. Find the length of the hypotenuse.
- 50. Draw an isosceles triangle with base of 16 units and height of 6 units. Find the length of the legs.

**Transformations** Name the type of transformation. *(Review 7.1–7.3, 7.5 for 8.7)*

- 51.
- 52.
- 53.